Math 10A
Midterm II; Thursday, 7/19/2018
Time: 2:10 PM **Name:** ______________________
Instructor: Roy Zhao **Student ID:** ______________________

- **DO NOT OPEN THE MIDTERM UNTIL TOLD TO DO SO!**
- Do all problems as best as you can. The exam is 80 minutes long. You may not leave during the last 30 minutes of the exam.
- Use the provided sheets to write your solutions. You may use the back of each page for the remainder of your solutions; in such a case, put an arrow at the bottom of the page and indicate that the solution continues on the back page. **No extra sheets of paper can be submitted with this exam!**
- The exam is closed notes and book, which means: **no class notes, no review notes, no textbooks, and no other materials can be used during the exam.** You can only use your cheat sheet. The cheat sheet is one side of one regular 8×11 sheet, handwritten.
- **NO CALCULATORS ARE ALLOWED DURING THE EXAM!**
- Justify all your answers, include all intermediate steps and calculations, and box your answers.

1. (22 points) Calculate the following integrals and derivatives.

   (a) (4 points) $\int e^{2x} dx =$

   (b) (5 points) $\int_{-5}^{5} \frac{\sin(x)}{x^4 + 3x^2 + 1} =$

   (c) (6 points) $\int_0^{\sqrt{\pi/2}} x \cos(x^2) dx =$

   (d) (7 points) $\frac{d}{dx} \int_x^{x^3} \frac{t \sin(t)}{e^t} dt =$

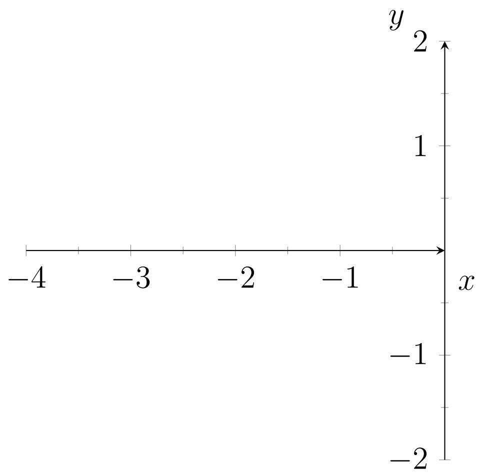
2. (16 points) (a) (12 points) Calculate $\displaystyle\int \frac{x^2+1}{x^2-1}dx$.

(b) (4 points) Set up the partial fractions decomposition of $\dfrac{3x^2+2x-4}{(x+1)(x^2-1)(x^2+1)^3}$.
(you do not need to solve for the constants)

3. (16 points) Integrate $\int e^x \cos(2x)dx$.

Answer: $\int e^x \cos(2x)dx =$

4. (22 points) (a) (10 points) Use the Trapezoid method with $n = 2$ to integrate $\displaystyle\int_{-3}^{-1} \frac{1}{x} dx$.
Sketch the function as well as what area your approximation calculates.



(b) (4 points) Without calculating the integral, is this an overestimate or underestimate?

(c) (8 points) Without calculating the integral, is this approximation within 0.5 of the actual answer?

5. (16 points) (a) (8 points) Calculate $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}dx$.

(b) (8 points) Does $\displaystyle\int_e^\infty \frac{\cos^2(x)}{(x\ln x)^2 + e^{-x^2}}dx$ converge?

6. (8 points) Bubble True or False. (1 point for correct answer, 0 if incorrect)

(a) Ⓣ Ⓕ We can only split an integral along its interval as in $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ only when c is between a and b.

(b) Ⓣ Ⓕ $\displaystyle\int_0^1 f'(x)\,dx = f(1) - f(0).$

(c) Ⓣ Ⓕ Suppose that $f'''(x) = 5$ for all $x \in [a, b]$. Then, Simpson's rule computes $\int_a^b f(x)\,dx$ exactly.

(d) Ⓣ Ⓕ Assume that $f(x) \geq 0$. In order to show that the integral $\displaystyle\int_1^\infty \frac{1}{f(x)}dx$ converges, it suffices to find a function $g(x)$ such that $f(x) \geq g(x) \geq 0$ on $[1, \infty)$ and show that $\displaystyle\int_1^\infty \frac{1}{g(x)}dx$ converges.

(e) Ⓣ Ⓕ $\displaystyle\int_{-1}^2 \frac{dx}{x} = \ln|x|\Big|_{-1}^2 = \ln 2 - \ln 1.$

(f) Ⓣ Ⓕ $\displaystyle\frac{d}{dx}\int_0^5 \sqrt{1-t}dt = \sqrt{1-x}.$

(g) Ⓣ Ⓕ If $f'(x) \leq g'(x) \leq 0$ for all $x \in [a, b]$, the error bound for using the left endpoint method to calculate $\displaystyle\int_a^b f(x)dx$ will be larger than for $\displaystyle\int_a^b g(x)dx$.

(h) Ⓣ Ⓕ The midpoint method will overestimate the integral $\displaystyle\int_0^1 x^3dx$.